

西南交通大学 2002 年硕士研究生招生入学考试

材料力学

试题

考试时间：2002 年 1 月

考生请注意：

1. 本试题共 七 题，共 12 页，考生请认真检查；
2. 答题时，直接将答题内容写在试题卷上；
3. 本试题不得拆开，拆开后果自负。

题号	一	二	三	四	五	六	七	八	九	总分
得分										
签字										

报考专业：

姓名：

考生编号：

请不要在虚线内答题

一、图示结构 C 结点与滑块铰接，不计滑块与滑槽间摩擦力，滑块只可沿滑槽上下自由移动，AB 与 BC 两杆面积相同且均为钢制，面积 $A=100\text{mm}^2$ ，材料拉压弹性模量 $E=2.0 \times 10^5 \text{MPa}$ ，线膨胀系数 $\alpha = 12 \times 10^{-6} (1/^\circ\text{C})$ 求当 BC 杆升温 50°C ，而 AC 杆温度不变时 C 处位移值。（15 分）

AC 杆内力 N_2 BC 杆内力 N_1

$$N_2 \cos 30^\circ = N_1$$

若无 AC 杆，则温度下 BC 自由伸长到 C_1 ，无温度应力

而 AC 杆约束，使 C 点回缩 Δl_2

此时 BC 杆内力 $N_1 = \frac{\Delta l_2 EA}{l_1}$

AC 杆内力 $N_2 = \frac{(\Delta l_1 - \Delta l_2) \cos 30^\circ EA}{l_2}$

$$l_2 \cos 30^\circ = l_1$$

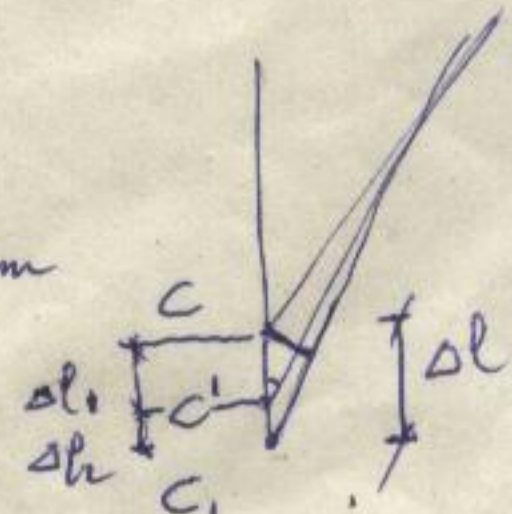
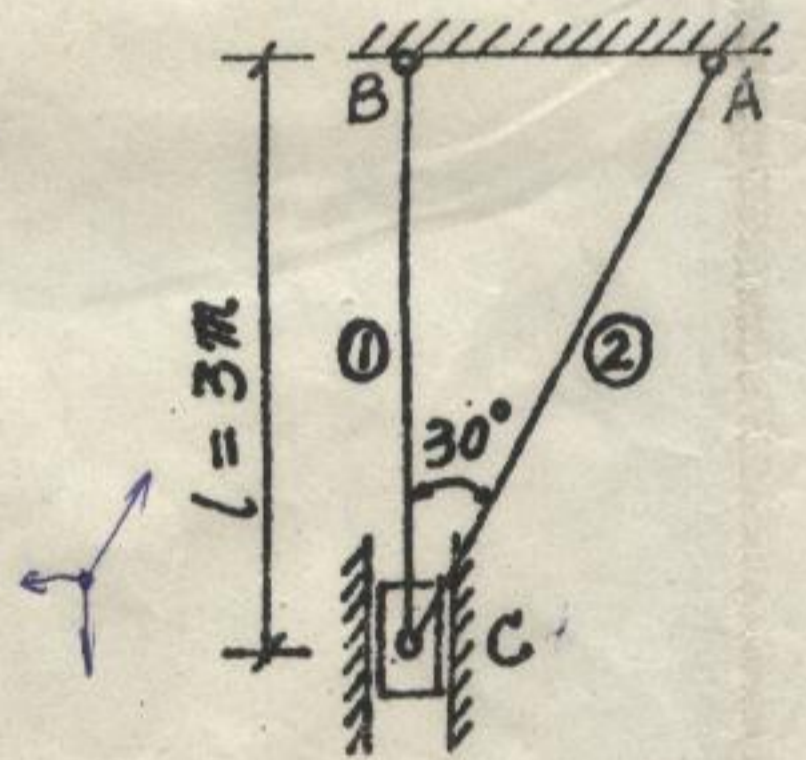
$$\frac{(\Delta l_1 - \Delta l_2) \cos 30^\circ EA}{l_2} \cos 30^\circ = \frac{\Delta l_2 EA}{l_2 \cos 30^\circ}$$

$$(\Delta l_1 - \Delta l_2) \cos^2 30^\circ = \Delta l_2$$

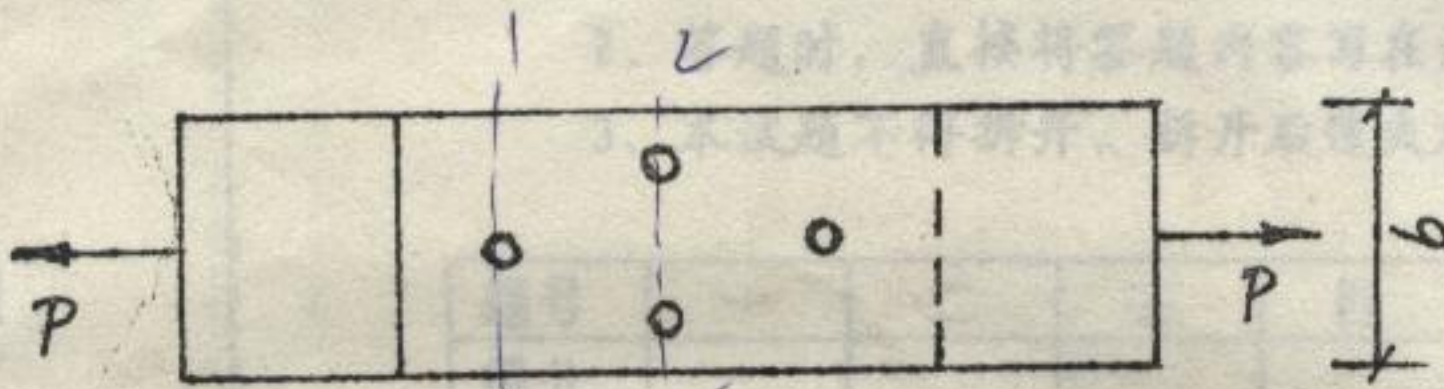
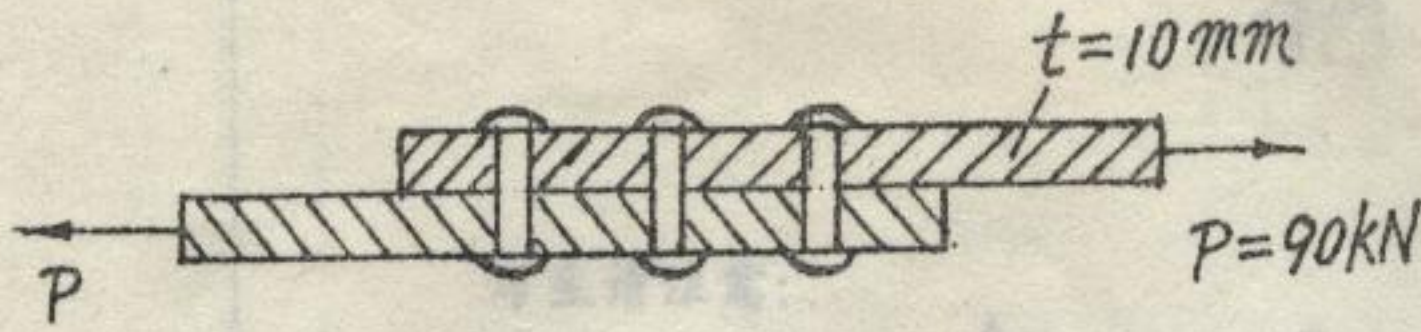
$$\Delta l_1 = \alpha \Delta t l_1 = 1800 \times 10^{-6} \text{ m} = 1.8 \text{ mm}$$

$$(1.8 - \Delta l_2) \frac{3\sqrt{3}}{8} = \Delta l_2$$

$$\Delta l_2 = 0.708 \text{ mm}$$



二、一拉杆接头如图所示，板厚 $t=10\text{mm}$ ，已知板与铆钉材料许用剪应力 $[\tau]=120\text{MPa}$ ，许用挤压应力 $[\sigma_c]=340\text{MPa}$ ，许用拉应力 $[\sigma]=160\text{MPa}$ ，试设计铆钉直径 d 及板宽 b 值。(15分)



力通过铆钉的几何中心，故设计时忽略偏心。

三个铆钉剪力

$$\tau = \frac{Q}{A} = \frac{90 \times 10^3}{3 \times \frac{\pi d^2}{4}} \leq [\tau] = 120 \text{ MPa}$$

$Q = P/4$
 $Q = 2P/4$
 $d \geq \sqrt{\frac{90 \times 10^3}{3 \times 120}} = 15.45 \text{ mm}$
 $d \geq 120$

铆钉挤压应力

$$\sigma_c = \frac{P/4}{td} = \frac{90 \times 10^3}{4 \times 10 d} \leq [\sigma_c] = 340 \text{ MPa}$$

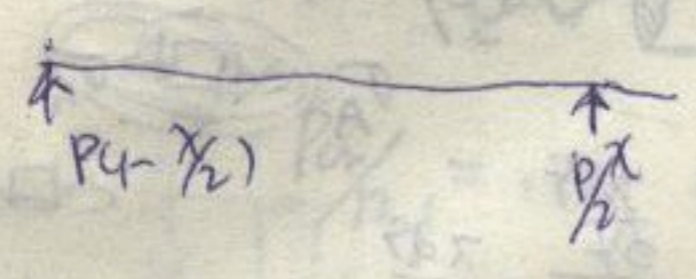
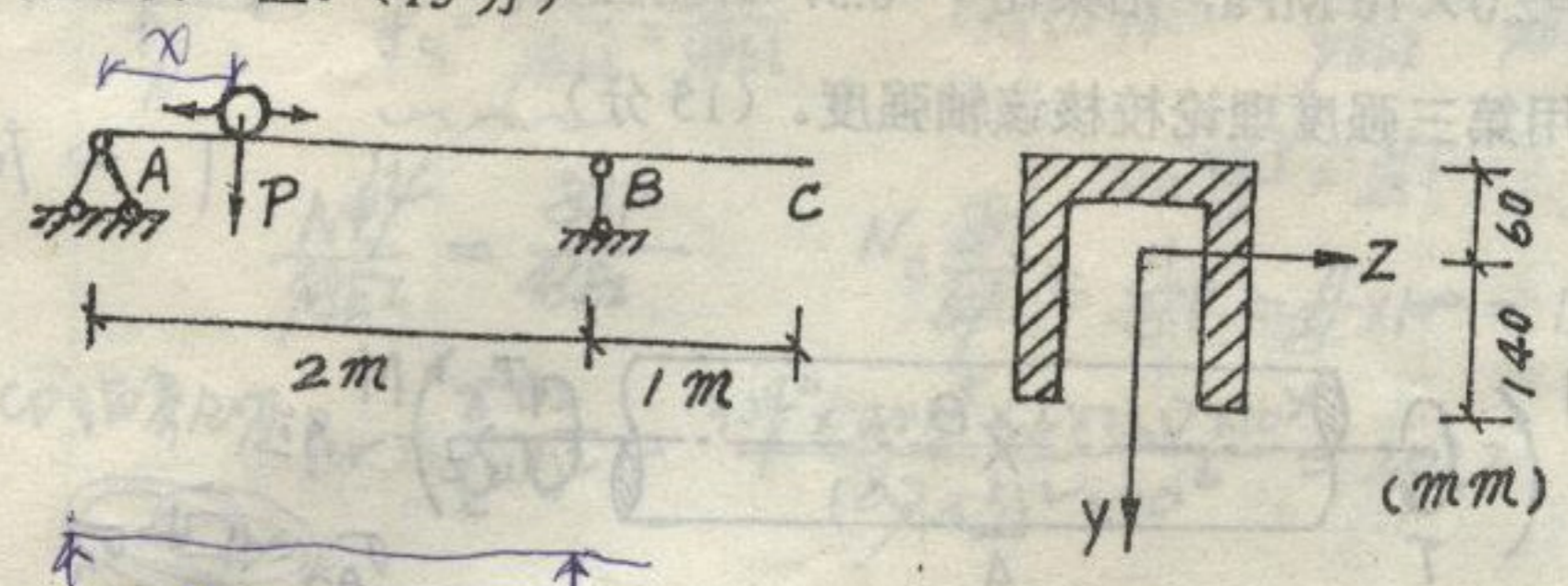
$$d \geq \frac{90 \times 10^3}{40 \times 340} = 66 \text{ mm}$$

与接板拉应力

$$\sigma_c = \frac{P}{bt - 2dt} = \frac{90 \times 10^3}{(b - 2d)t} \leq [\sigma] = 160 \text{ MPa}$$

$d \geq 20 \text{ mm}$
 $b \geq 100 \text{ mm}$

四、图示外伸梁上荷载 P 可沿梁水平移动，梁截面为槽形， $I_z = 4.0 \times 10^7 \text{mm}^4$ ，梁材料许用拉应力 $[\sigma_{拉}] = 35 \text{MPa}$ ，许用压应力 $[\sigma_{压}] = 140 \text{MPa}$ ，求该梁的容许荷载 P 值。(15分)



P 在 AB 中点 (1m) 处 $M \uparrow$ $\sigma = \frac{M}{I} y$

$$\sigma_{拉} = \frac{\frac{P}{2} \times 1000}{4 \times 10^7} \times 140 = 1.75P \leq [\sigma_{拉}] = 35 \text{ MPa}$$

$$\sigma_{压} = \frac{\frac{P}{2} \times 1000}{4 \times 10^7} \times 60 = 0.75P \leq [\sigma_{压}] = 140 \text{ MPa}$$

P 在 C 处 B 点 $M \uparrow$

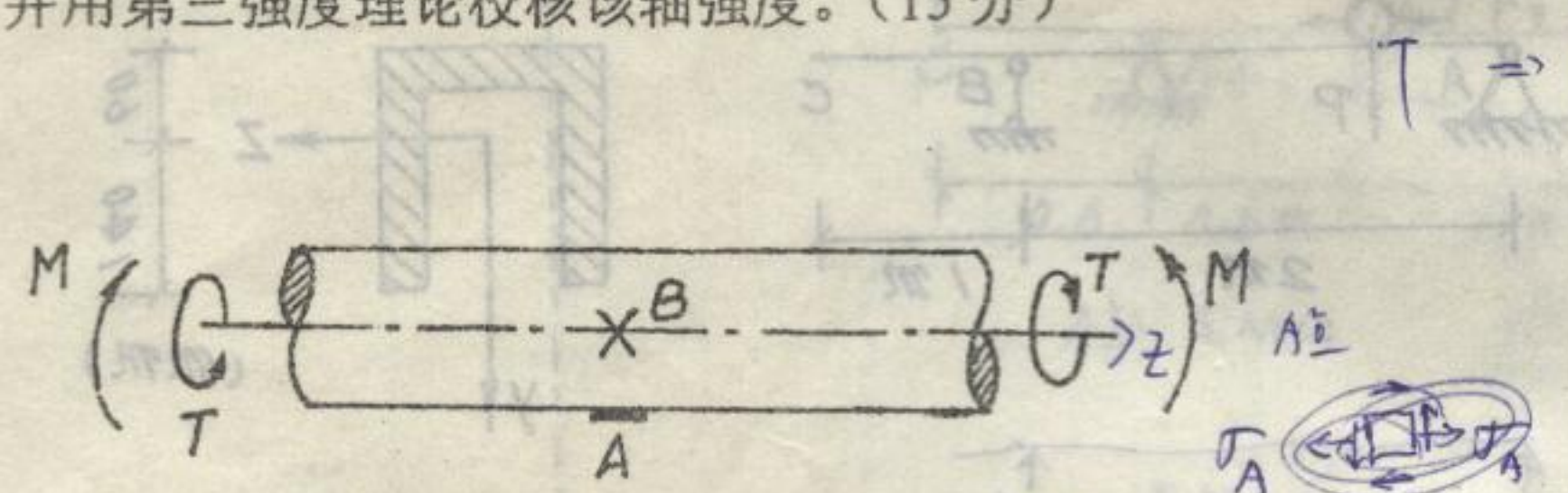
$$\sigma_{拉} = \frac{P \times 1000}{4 \times 10^7} \times 60 = 1.5P \leq [\sigma_{拉}] = 35 \text{ MPa}$$

$$\sigma_{压} = \frac{P \times 1000}{4 \times 10^7} \times 140 = 3.5P \leq [\sigma_{压}] = 140 \text{ MPa}$$

- $P \leq 20 \text{ kN}$
- $P \leq 18 \text{ kN}$
- $P \leq 23 \text{ kN}$
- $P \leq 40 \text{ kN}$

容许荷载 20 kN

五、图示钢制圆轴受弯矩 M 及扭矩 T 联合作用，圆轴直径 $d=18\text{mm}$ ，
 现测得圆轴表面最低 A 点处沿轴向线应变 $\epsilon_0 = 5.0 \times 10^{-4}$ ，测得水平轴切面上 B 点处 45° 方向上线应变 $\epsilon_{45^\circ} = 4.2 \times 10^{-4}$ 及 $\epsilon_{-45^\circ} = -4.2 \times 10^{-4}$ ，已知钢弹性模量 $E=2.0 \times 10^5 \text{MPa}$ ，泊桑比 $\nu=0.3$ ，许用应力 $[\sigma]=170 \text{MPa}$ ，试求 M 及 T 值，并用第三强度理论校核该轴强度。(15分)



$T \Rightarrow$ 右端为左旋，离开截面为已

$A \dot{z}$: $\sigma_A = \frac{M}{I W_z}$ $W_z = \frac{I_z}{d/2} = \frac{\frac{\pi d^4}{64}}{d/2} = \frac{\pi d^3}{32}$

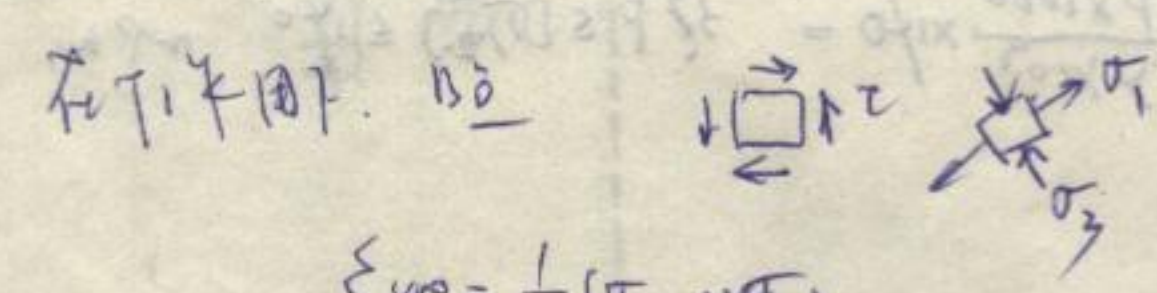
$A \dot{z}$ 处沿轴向应变 $\epsilon_0 = \frac{\sigma_A}{E} \Rightarrow \epsilon_0 E = \frac{M}{W_z}$

$$\begin{cases} I_z = \frac{\pi d^4}{64} \\ I_T = \frac{\pi d^4}{32} \\ W_z = \frac{\pi d^3}{32} \\ W_p = \frac{\pi d^3}{16} \end{cases}$$

$$M = \epsilon_0 E W_z = 5.0 \times 10^{-4} \times 2.0 \times 10^5 \times \frac{\pi \times 18^3}{32}$$

$$\begin{cases} \sigma_1 - \sigma_3 = \epsilon_0 E = 5.0 \times 10^{-4} \times 2.0 \times 10^5 = 100 \text{ MPa} < [\sigma] = 170 \text{ MPa} \\ \sigma_1 = \sigma_A \\ \sigma_3 = 0 \end{cases}$$

$B \dot{z}$: 在水平切面上. $B \dot{z}$ 处在水平切面上 $\sigma_x = 0$ $\tau_y = \frac{QS}{bZ}$



$$\epsilon_{45^\circ} = \frac{1}{E} (\sigma_1 - \nu \sigma_3)$$

$$E \epsilon_{45^\circ} = T \left(\frac{1}{W_p} + \frac{\nu}{W_p} \right)$$

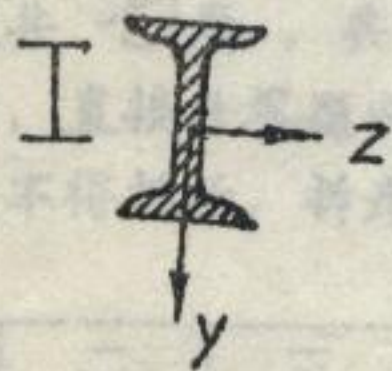
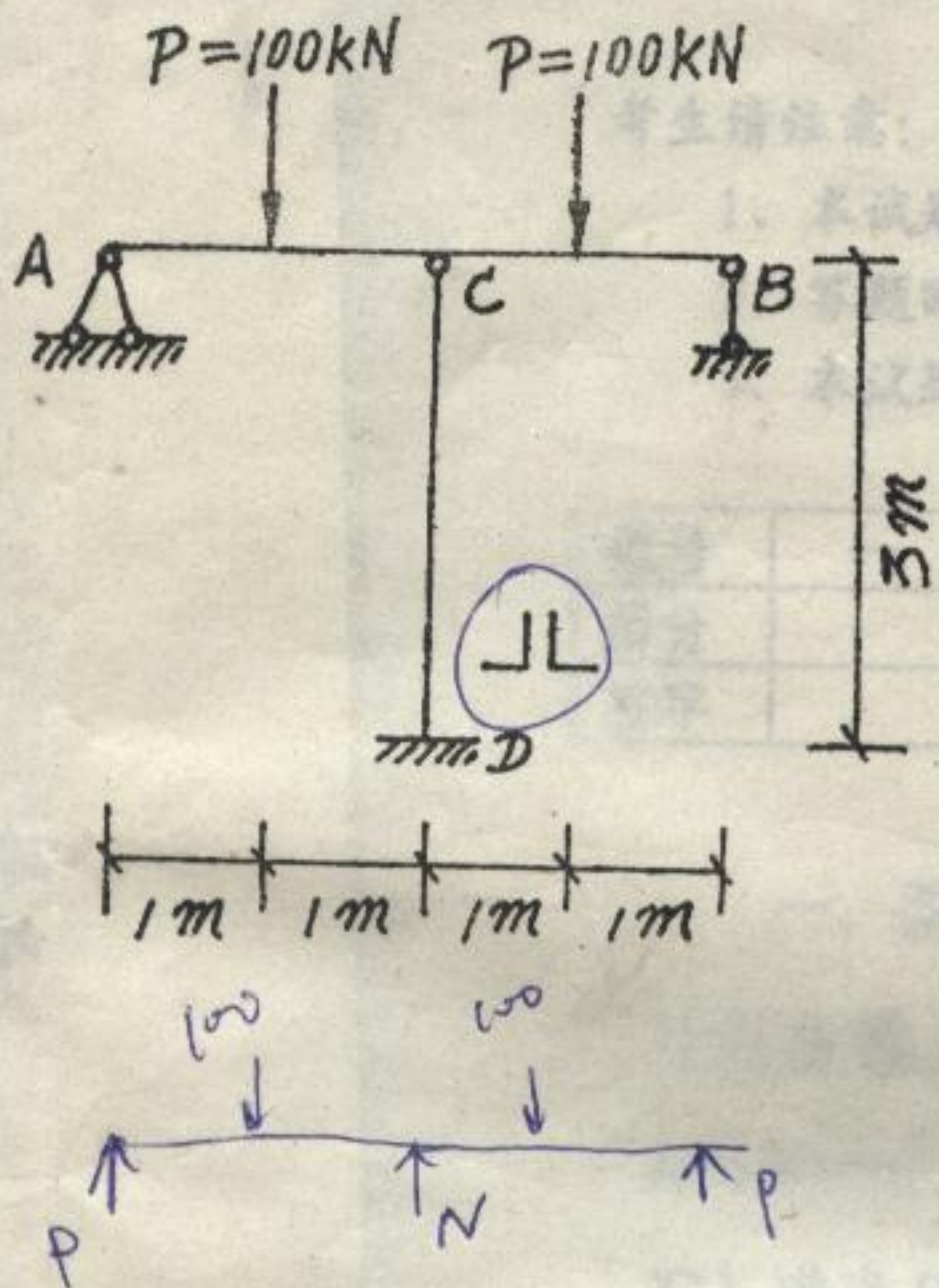
$$T = \epsilon_{45^\circ} E W_p / (1 + \nu) = 4.2 \times 10^{-4} \times 2.0 \times 10^5 \times \frac{\pi \times 18^3}{16} / (1 + 0.3) =$$

$$\sigma_1 - \sigma_3 = \frac{2T}{W_p} = \frac{2 \times 4.2 \times 10^{-4} \times 2.0 \times 10^5 \times \frac{\pi \times 18^3}{16}}{2 \times 10^5 \times \frac{\pi \times 18^3}{16} / (1 + 0.3)} = \frac{168}{1.3} = 129.2 \text{ MPa} < [\sigma] = 170 \text{ MPa}$$

$$\begin{cases} I_p = \frac{M_p \rho}{I_p} \\ T_{max} = \frac{M_p}{W_p} \end{cases}$$

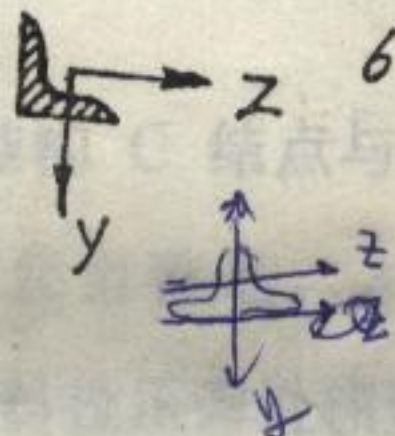
$$\tau = \frac{T}{W_p} = \frac{T}{\frac{\pi d^3}{16}}$$

六、梁 AB 为 16# 工字钢，立柱 CD 为两根 6.3# 等边角钢拼装而成，立柱与梁联接处为铰接，立柱下端为固定端，已知材料弹性模量 $E=2.0 \times 10^5 \text{MPa}$ ，比例极限 $\sigma_p = 190 \text{MPa}$ ，取稳定安全系数 $n_w = 1.8$ ，若不计 CD 柱压缩变形量，试校核 CD 立柱稳定性。(15 分)



16# 工字钢 $I_z = 11.30 \times 10^6 \text{mm}^4$
 $W_z = 141 \times 10^3 \text{mm}^3$

P210



6.3# 角钢 $I_y = I_z = 23.17 \times 10^4 \text{mm}^4$
 $A = 614.3 \text{mm}^2$
 $i_y = i_z = 19.4 \text{mm}$

拼装后为圆形 $I_z = 2I_{z1} = 2 \times 23.17 \times 10^4 \text{mm}^4$
 $A = 2A_1 = 614.3 \times 2 \text{mm}^2$
 $i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{2I_{z1}}{2A_1}} = i_{z1} = 19.4 \text{mm}$

立柱 CD 的长细比为 λ

即 $\lambda = \frac{\mu l}{i}$

$$\lambda = \frac{\mu l}{i_z} = \frac{0.7 \times 3000}{19.4} = 108 > \lambda_p$$

即 $\lambda > \lambda_p$ 属于大长细比

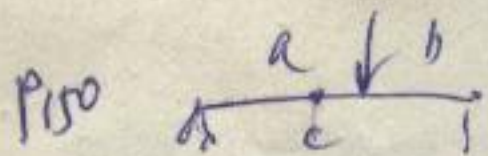
$$\sqrt{\frac{\pi^2 E}{\sigma_p}} = \sqrt{\frac{\pi^2 \times 2 \times 10^5}{190}} = 100 \sqrt{\frac{2 \times 10^5}{190}} = 101$$

$$\begin{cases} \sigma = \frac{P_{cr}}{A} \\ P_{cr} = \frac{\pi^2 E I}{(\mu l)^2} \end{cases}$$

$$P_{cr} = \frac{\pi^2 E I}{(\mu l)^2}$$

$$= \frac{\pi^2 E}{\left(\frac{\mu l}{i}\right)^2} = \frac{\pi^2 E}{\lambda^2} \leq \sigma_p$$

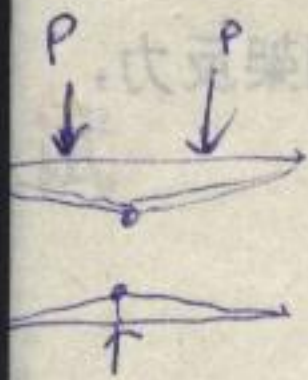
$$\lambda \geq \sqrt{\frac{\pi^2 E}{\sigma_p}}$$



$$f_3 = \frac{Pb(3l^2 - 4b^2)}{48EI}$$

$$f_1 = \frac{P(3l^2 - 4a^2)}{48EI}$$

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$$f_4 = \frac{Pl}{48EI} = \frac{64N}{48EI}$$

$$\frac{64N}{48EI} = \frac{88P}{48EI}$$

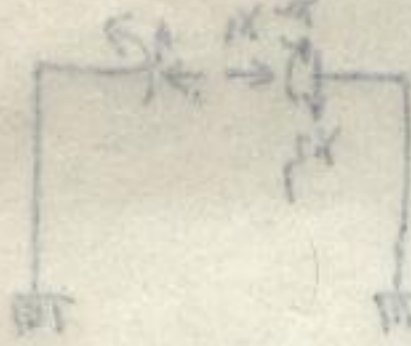
$$N = \frac{88P}{64} = \frac{11P}{8}$$

$$N = \frac{11}{8}P = \frac{11}{8} \times 100 = 137.5 \text{ kN}$$

$$P_{cr} = \frac{\pi^2 EI}{(l_{eff})^2} = \frac{\pi^2 \times 2 \times 10^5 \times 2 \times 23.7 \times 10^4}{(0.7 \times 3)^2 \times 10^6} = 207.2 \text{ kN}$$

$$\frac{P_{cr}}{m} = 115.1 \text{ kN}$$

$$N > [P] = \dots$$



Handwritten calculations for a system of equations:

$$\begin{aligned} 3x + 5y + 9z &= 20 \\ 9x + 5y + 9z &= 20 \\ 9x + 5y + 9z &= 20 \end{aligned}$$

Subtracting the first equation from the second and third:

$$\begin{aligned} 6x &= 0 \implies x = 0 \\ 6y &= 0 \implies y = 0 \\ 6z &= 0 \implies z = 0 \end{aligned}$$

七、用能量方法(单位力法或卡氏第二定理)求解图示超静定刚架反力, 刚架抗弯刚度 EI 已知, 不计剪切与轴向变形影响。(10分)

假设超静力 C 点位移为 x_1

$$\frac{\partial U}{\partial x_1} = \frac{\partial}{\partial x_1} \int_0^L (x_2 - py + x_1 y) dy$$

$$= \frac{\partial}{\partial x_1} \left(\frac{x_2 L^2}{2} - \frac{pL^3}{6} + \frac{x_1 L^2}{2} \right)$$

$$\frac{\partial U}{\partial x_2} = \frac{\partial}{\partial x_2} \left(\int_0^L x_2 dy + \int_0^L (x_2 - py) dy \right)$$

$$= \frac{\partial}{\partial x_2} \left(\frac{x_2 L}{2} + x_2 L - \frac{pL^2}{2} + \frac{x_1 L^2}{2} \right)$$

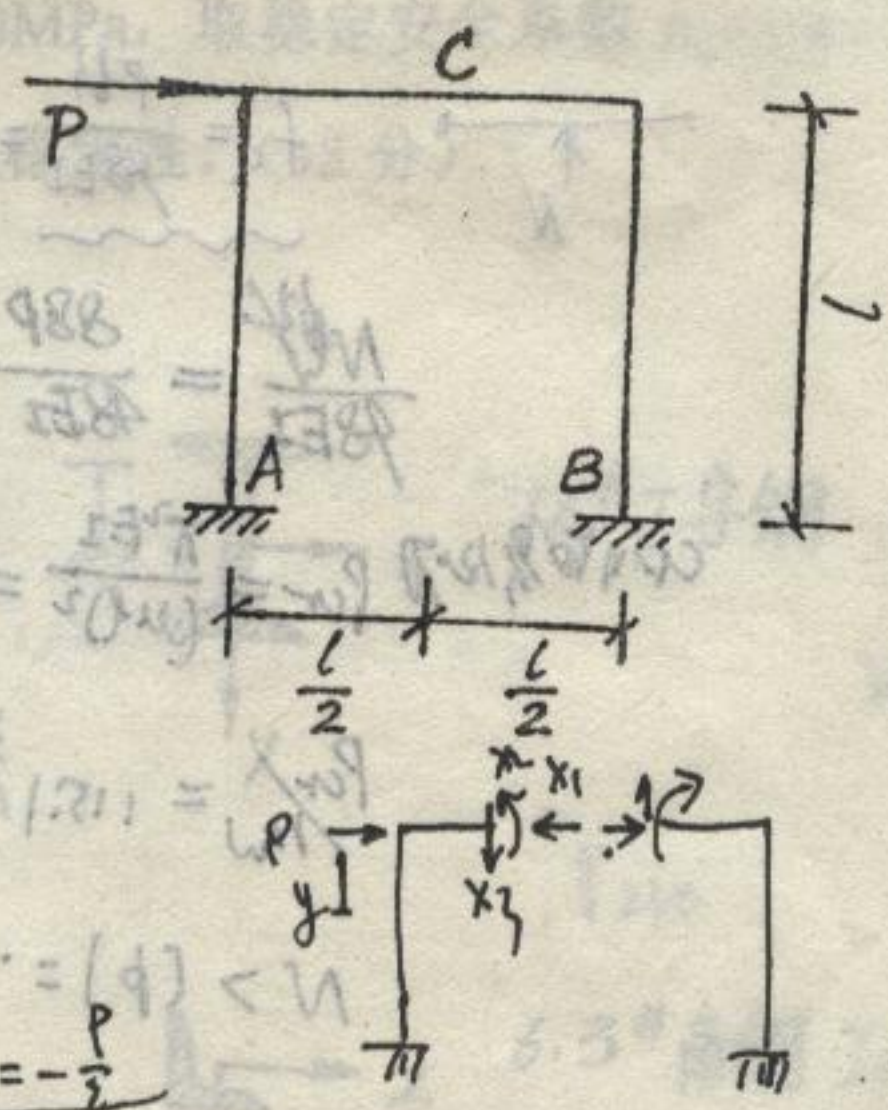
$$\frac{\partial U}{\partial x_1} = 0 \quad \frac{\partial U}{\partial x_2} = 0$$

$$\begin{cases} 3x_2 = 2pL + 2x_1 l \\ 3x_2 l = \frac{pL^2}{2} - \frac{x_1 l^2}{2} \end{cases}$$

$$\begin{cases} 3x_2 = 2pL + 2x_1 l \\ 3x_2 = \frac{pL^2}{2l} - \frac{x_1 l}{2} \end{cases}$$

$$\begin{cases} 3x_2 - 2pL + 2x_1 l = 0 \\ 3x_2 - \frac{pL^2}{2l} + \frac{x_1 l}{2} = 0 \end{cases}$$

$$x_1 = \frac{pL}{3}$$



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